

PROCESS REGION CHARACTERISTICS AND STABLE CRACK GROWTH

Per Ståhle



LUND INSTITUTE OF TECHNOLOGY
DIVISION OF SOLID MECHANICS
BOX 118
S - 221 00 LUND, SWEDEN
1985



PROCESS REGION CHARACTERISTICS AND STABLE CRACK GROWTH

P. STAHLE

Division of Solid Mechanics, Lund Institute of Technology, Lund, Sweden

ABSTRACT

An investigation of stable crack growth is carried out assuming a line shaped process region. The incipient and continued development of the process region up to instability and the steady-state case are investigated. Different hardening rates and different decohesive properties of the process region are considered. At large cracks extensive stable crack growth precedes global instability, whereas such crack growth is rather limited at smaller cracks.

1. Introduction

The process region near a crack tip is generally neglected or assumed to be point-sized in analytical and numerical analyses of fracture. This simplifies the treatment considerably but gives rise to several disadvantages. One is a very unrealistic description of the near-tip region, because continuum behaviour is assumed and extrapolation to infinite strains is needed. Another one, appearing in connection with moving cracks and vanishing strain hardening near the crack tip, is difficulties to obtain a reasonable energy flow to the crack tip, cf. Broberg [1].

These disadvantages can be avoided if the finite size of the process region is considered. In reality the unstable material behaviour, characteristic of the process region is spread over a volume ahead of the crack tip. Here, however, this volume is assumed to be degenerated to a line shaped region straight ahead of the crack tip.

The line model of the process region, originally introduced by Barenblatt [2], was employed by Andersson [3]. He simulated crack growth in finite element analyses, through a nodal release technique for a mode I crack in plane stress. His results revealed the importance of the element size when

crack tip opening angle or displacement is adopted for a fracture criterion. The same results has later been obtained by other investigators, for instance Kfouri and Miller [4]. An approximate analytical solution was found by Lundström [5] for a steady-state mode III crack. He investigated different decohesive properties and showed that the energy dissipation rate in the process region is negligible compared to the dissipation rate in the plastic zone for process regions that are small compared to the linear extension of the plastic zone. A pilot investigation [6] to the present paper, at which plasticity off-side the process region was neglected, resulted in the occurrence of global instability before the process region was fully developed at small cracks, whereas global instability occurred for large cracks when the process region was fully developed.

2. The problem

Consider a large plate of an elastic-plastic linearly hardening material, described by Young's modulus E , Poisson's ratio ν , yield stress σ_Y and hardening rate $H = d\sigma_e/d\varepsilon_e^P$ at plastic flow, where σ_e and ε_e^P are effective stress and effective plastic strain. Plastic flow is governed by von Mises yield criterion and the associated flow rule.

A straight, through thickness, crack is situated at the centre of the plate. A coordinate system x, y is introduced such that the crack occupies the position $|x| < c, y = 0$ (see Fig. 1). The line shaped process regions are

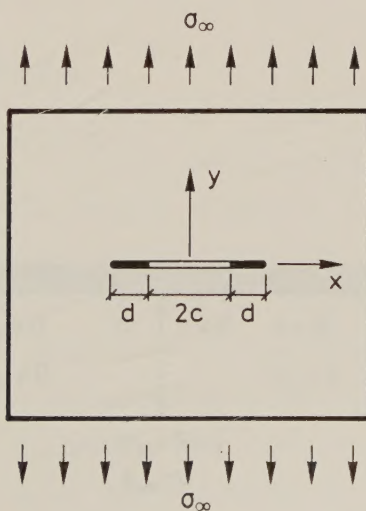


Figure 1. An infinite body with a crack.

assumed to be symmetrically situated at $c \leq |x| < c + d$, $y = 0$, ahead of both crack tips. The process regions are characterized by a displacement discontinuity, $2v$, in the y -direction, and a boundary stress σ_y monotonically decreasing with increasing displacement v . A remotely applied stress σ_∞ is acting in the y -direction. Plane strain conditions are assumed throughout. Due to the symmetry with respect to $x = 0$ and with respect to $y = 0$ only one quarter, $x \geq 0$, $y \geq 0$ of the plate is considered. Figure 2 shows the boundary conditions:

$$\sigma_y = 0 \quad \text{for } 0 \leq x < c, \quad y = 0 \quad (1)$$

$$\sigma_y = s(v) \quad \text{for } c \leq x < c + d, \quad y = 0 \quad (2)$$

$$\tau_{xy} = 0 \quad \text{for } 0 \leq x, \quad y = 0 \quad (3)$$

$$v = 0 \quad \text{for } c + d \leq x, \quad y = 0 \quad (4)$$

$$\tau_{xy} = 0 \quad \text{for } x = 0, \quad 0 \leq y \quad (5)$$

$$\text{and } u = 0 \quad \text{for } x = 0, \quad 0 \leq y \quad (6)$$

where u and v are the displacements in the x - and y -directions and $s(v)$ is a function describing the properties of the process region. In the present investigation it is assumed that

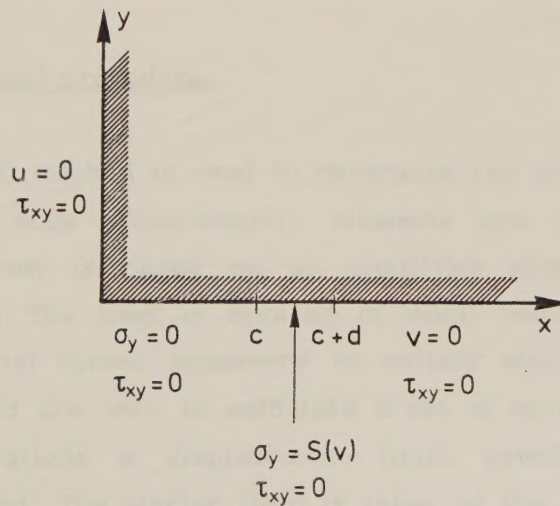


Figure 2. Boundary conditions for one quarter of the body.

$$s(v) = \sigma_D (1 - v/v_*) \quad (7)$$

where σ_D is the maximum stress at the process region boundary and v_* the largest possible displacement before complete decohesion (see Fig. 3).

The length d of the process region cannot be specified a priori, but follows from the requirement that σ_y must be continuous at $x = c+d$. c is the current half length of the crack. Before crack growth $c = c_0$, where c_0 is fixed.

Four different sets of material parameters are considered

$$A: \begin{cases} \sigma_D/\sigma_Y = 2.5 \\ H/E = 0.01 \end{cases} \quad B: \begin{cases} \sigma_D/\sigma_Y = 2.5 \\ H/E = 0.001 \end{cases} \quad C: \begin{cases} \sigma_D/\sigma_Y = 3 \\ H/E = 0.01 \end{cases} \quad D: \begin{cases} \sigma_D/\sigma_Y = 3 \\ H/E = 0.001 \end{cases}$$

For all four materials Poisson's ratio $\nu = 0.3$.

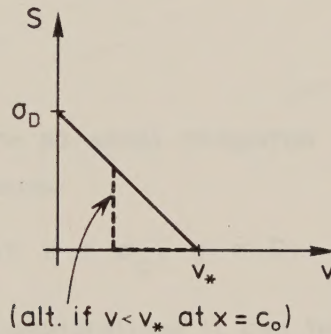


Figure 3. Decohesive stress versus discontinuous displacement at the process region. The dotted part shows $s(v)$ for a situation when the displacement v at the crack tip is less than the largest possible displacement v_* at the process region.

3. The finite element procedure.

The finite element method is used to determine the stress and strain fields. Quadrilateral 8 node isoparametric elements are used throughout. The non-linear analyses is based on an algorithm proposed by Nayak and Zienkiewicz [7]. The load is applied in small increments. At each load increment, residual forces necessary to satisfy equilibrium for a starter displacement field are used to calculate a set of corrective displacements. After some iterations a displacement field, satisfying the equilibrium conditions is found. The starter field is taken as the resulting displacement field at the previous increment. However, the starter field at the first

increment is taken to be the one found by assuming linear elasticity throughout.

The processes in the process region are modelled by a slowly progressing node release. A node at $x > c_0$, $y = 0$ is included in the process region as the nodal force F_0 in the y -direction reaches a value F_D consistent with the stress σ_D . This node is then released according to (7) until $F_0 = 0$, i.e. the node leaves the process region and is then a part of the crack surface.

Special attention is given to two cases in which the load can be applied in one single step. These cases are stationary cracks at such a small load that self-similarity can be assumed for the developing plastic region and growing cracks for which a steady-state can be assumed in the crack tip vicinity. For both these cases the pertinent strain history is included in each current strain field solution. Thus the method is modified according to [8], for these cases.

4. Self-similarity

Consider loads that are so small compared to the load at incipient crack growth that one may assume

$$v \ll v_* \quad \text{at } x = c_0, y = 0 \quad (8)$$

Then by dimensional considerations it can be shown that the situation near the crack tip is characterized by an approximate self-similarity. The stress intensity factor K_I is defined for the present (small scale yielding) case by

$$K_I = \sigma_\infty \sqrt{\pi c_0} \quad (9)$$

Now a polar coordinate system with $r = 0$ at $x = c_0$, $y = 0$ (the crack tip) and $\theta = \pm\pi$ on the crack surfaces is introduced. A half-circular region $r \leq a$, $0 \leq \theta \leq \pi$ is considered. Assume that σ_∞/σ_Y is small enough that

$$(K_I/\sigma_Y)^2 \ll a \ll c_0 \quad (10)$$

Then the load on the surface $r = a$ of the half-cylinder can be described by the dominating stress terms of a Williams expansion [9] around $x = x_0 = c_0 + (K_I/\sigma_Y)^2/(6\pi)$, $y = 0$, rather than around $x = c_0$, $y = 0$, following a recommendation [10] for a related case. The boundary conditions on $r = a$, $0 \leq \theta \leq \pi$ are

$$\sigma_r = \frac{K_I}{\sqrt{2\pi r_0}} \left[1 - \sin(\theta_0/2) (\sin(3\theta_0/2) \cos 2\theta - \cos(3\theta_0/2) \sin 2\theta) \right] \cos(\theta_0/2) \quad (11)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r_0}} \left[\sin(3\theta_0/2) \sin 2\theta + \cos(3\theta_0/2) \cos 2\theta \right] \cdot \frac{1}{2} \sin \theta_0 \quad (12)$$

where $r_0 = [(x-x_0)^2 + y^2]^{1/2}$ and $\theta_0 = \tan^{-1}[y/(x-x_0)]$. Further the stress at the boundary of the process region, at $0 \leq r < d$, $\theta = 0$ equals

$$\sigma_\theta = \sigma_D \quad (13)$$

and the remaining boundary conditions are

$$\sigma_\theta = 0 \quad \text{on } r \leq a, \theta = \pi \quad (14)$$

$$\tau_{r\theta} = 0 \quad \text{on } r \leq a, \theta = 0 \text{ and } \theta = \pi \quad (15)$$

$$u_\theta = 0 \quad \text{on } d \leq r \leq a, \theta = 0 \quad (16)$$

The area $r \leq a$, $0 \leq \theta \leq \pi$ is covered by 96 elements as shown in Fig. 4. The finite element code used is adopted to self-similarity problems i.e. the strain history is traced along rays ($\theta = \text{const.}$) diverging radially from the crack tip. The load is applied in one single step and iterations are performed

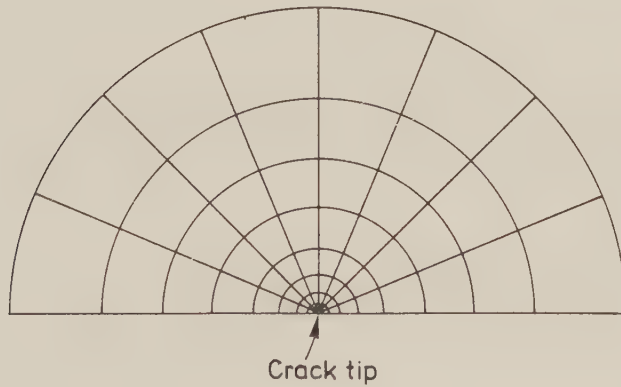


Figure 4. A principal representation of the element mesh for the self-similar problem.

for equilibrium.

A certain length d' (a few elements) of the process region is assumed beforehand. Hence the required continuity for σ_θ at $r = d'$, $y = 0$ cannot in general be satisfied for $\sigma_\theta = \sigma_D$, but rather for some other value σ_D' . Then, repeatedly, new problems are solved for which the yield stress is put equal to $\sigma_Y' = \sigma_D' \sigma_Y / \sigma_D$ where σ_D' is taken as the result from the previous problem. This procedure is interrupted when two consecutive values of σ_Y' become sufficiently close. This implies that the correct ratio $\sigma_D' / \sigma_Y' = \sigma_D / \sigma_Y$ is arrived at. Hence the solution of the original problem is arrived at by scaling, i.e. $d = (\sigma_Y' / \sigma_Y)^2 d'$ and $v = (\sigma_Y' / \sigma_Y) v'$ where v is the displacements at the crack tip and v' its value before scaling.

Results for the different materials, shown in Table 1, can be compared to the results for the Dugdale model [11]. The effective stress immediately outside the Dugdale zone, at a large crack and uniaxial stress at infinity, is $(1-2\nu)\sigma_D = 0.4\sigma_D$. By putting $\sigma_Y = 0.4\sigma_D$ (as for materials A and B) one obtains $\sigma_Y^2 d / K_I^2 = 0.0627$ and $\nu E \sigma_Y / [(1-\nu^2) K_I^2] = 0.2$ for the Dugdale model. These results can be used for a control of the accuracy of the numerical calculations. To this end the numerical procedure was applied on a linearly elastic material. The result turned out to be $\sigma_Y^2 d / K_I^2 = 0.0613$ and $\nu E \sigma_Y / [(1-\nu^2) K_I^2] = 0.205$ indicating an error of about 3%.

There is, as observed, very little difference between the materials as regards the crack tip opening displacement at the same remote load whereas the length of the process region varies considerably. For materials with $\sigma_D / \sigma_Y = 3$ the length of the process region is very small compared to the

Table 1.

Non-dimensional length $\sigma_Y^2 d / K_I^2$, of the process region and non-dimensional crack opening displacement $\nu E \sigma_Y / [(1-\nu^2) K_I^2]$ for the four different materials A, B, C, D considered.

	$\sigma_Y^2 d / K_I^2$	$\nu E \sigma_Y / [(1-\nu^2) K_I^2]$
A	0.046	0.20
B	0.040	0.22
C	0.013	0.19
D	0.008	0.20

linear extension of the plastic region and when σ_D/σ_Y is only slightly above 3 it disappears. Now from Table 1 one finds that the non-dimensional quantity $\nu E \sigma_D / [(1-\nu^2) K_I^2]$ is about 0.6 for the shortest process region (for material D). If one assumes that this value would not change appreciably if σ_D/σ_Y were increased beyond the value 3, then it might be representative even for a nearly point-sized process region, i.e. one whose length is considerably smaller than the linear extension of the plastic region. Then the J-integral for a path immediately outside the process region is given by:

$$J = 2 \int \sigma_D dv = 2 \sigma_D v \approx 1.2 (1-\nu^2) K_I^2 / E \quad (17)$$

This deviates considerably from the result for a point-sized process region by Levy et al. who found a value of the J-integral by numerical integration for a path surrounding the immediate vicinity of the crack tip:

$$J \approx 0.75 (1-\nu^2) K_I^2 / E \quad (18)$$

Examination of an analytical asymptotic solution for the non-hardening case might explain why d is considerably smaller for $\sigma_D = 3\sigma_Y$ than for $\sigma_D = 2.5\sigma_Y$. Thus it has been noted by Rice [12] that the Prandtl slip line field represents the limiting state for the elastic-perfectly-plastic material as the crack tip is approached. This solution shows that $\sigma_\theta = (2+\pi)/\sqrt{3} \sigma_Y \approx 2.96\sigma_Y$ ahead of the crack tip and a process region with a larger decohesive stress is therefore not allowed. The same conclusion ought to be approximately valid even at finite but low hardening rates, e.g. $H/E \leq 0.01$. Anyway, attempts to perform computations for $\sigma_D > 3\sigma_Y$ failed, because it was not possible to increase the load as much as would be necessary to develop an observable process region.

One observes that the hardening rate has rather little influence on the extension of the plastic zones (see Figs. 5 and 6), whereas different process region characteristics seem to be more critical. Thus the area of the plastic zone is about 50 per cent larger for $\sigma_D/\sigma_Y = 3$ than for $\sigma_D/\sigma_Y = 2.5$. The results for materials C and D are rather similar to the result of Levy et al. [13], who neglected the process region. One also observes that the process region is well embedded in the plastic region for materials C and D, i.e. for $\sigma_D/\sigma_Y = 3$ but that it almost pierces the plastic region for materials A and B, i.e. for $\sigma_D/\sigma_Y = 2.5$.

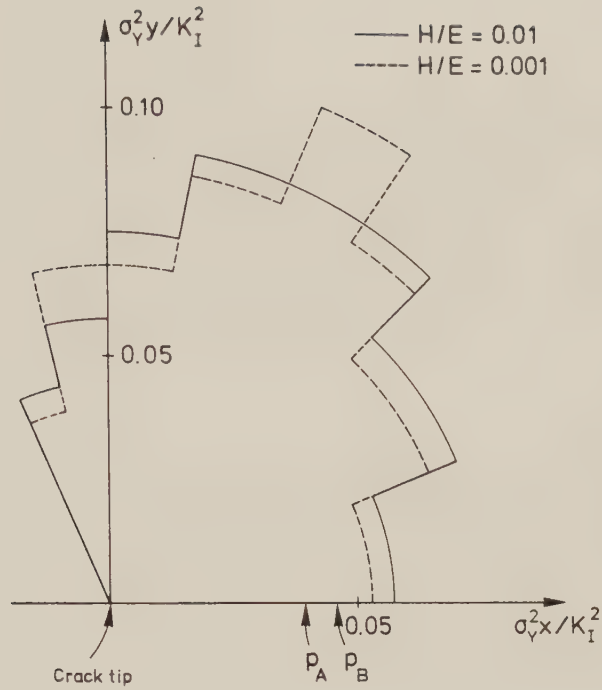


Figure 5. The shape of the plastic zones at self-similarity for materials A (full lines) and B (dotted lines) with $\sigma_D/\sigma_Y = 2.5$. The position of the tip of the process region is at p_A for material A and at p_B for material B.

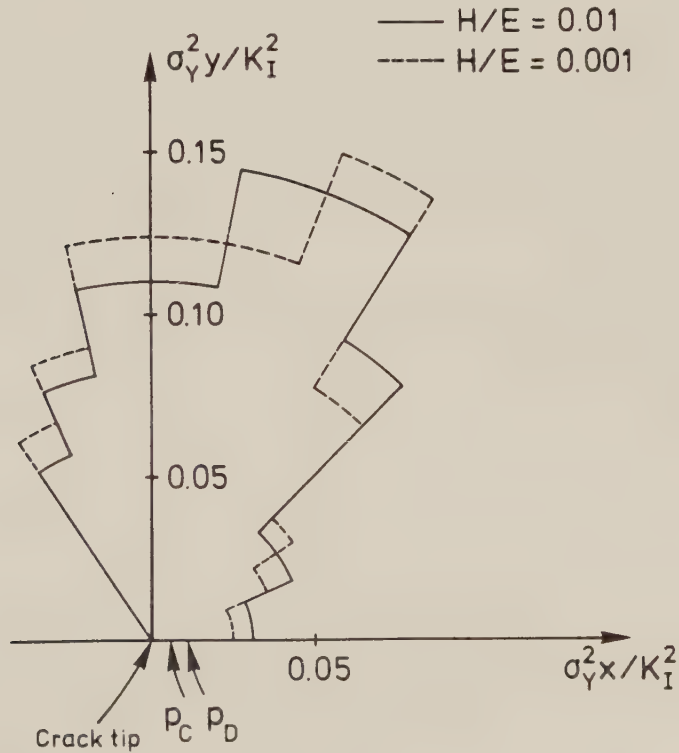


Figure 6. The shape of the plastic zones at self-similarity for materials C (full lines) and D (dotted lines) with $\sigma_D/\sigma_Y = 2.5$. The position of the tip of the process region is at p_C for material C and at p_D for material D.

5. Steady-state

For very large cracks a near-tip steady-state will be approached after some stable crack growth. The steady-state case can be investigated by introducing a moving coordinate system x', y , for which $x' = x - c$.

A region $-a \leq x' \leq a$, $0 \leq y \leq a$ is considered. a is assumed to be small compared to the crack length c . In fact inequalities

$$(K_I/\sigma_Y)^2 \ll a \ll c \quad (19)$$

are assumed to be satisfied. The load on the boundary surface is described by the square root singular stress terms of an expansion around $x' = x'_0 = (K_I/\sigma_Y)^2/(6\pi)$, $y = 0$. Thus the boundary conditions can be written:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r_0}} [\cos(\theta_0/2)(1+\sin(\theta_0/2)\sin(3\theta_0/2))] \quad (20)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r_0}} \cos(\theta_0/2)\sin(\theta_0/2)\sin(3\theta_0/2) \quad (21)$$

on $-a \leq x' \leq a$, $y = a$,

$$\sigma_x = \frac{K_I}{\sqrt{2\pi r_0}} [\cos(\theta_0/2)(1-\sin(\theta_0/2)\sin(3\theta_0/2))] \quad (22)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r_0}} \cos(\theta_0/2)\sin(\theta_0/2)\sin(3\theta_0/2) \quad (23)$$

on $x' = a$, $0 \leq y \leq a$ and on $x' = -a$, $p < y \leq a$ and

$$\partial u/\partial x = \partial u^e/\partial x \quad (24)$$

$$\partial v/\partial x = \partial v^e/\partial x \quad (25)$$

on $x' = -a$, $0 \leq y \leq p$.

Here $r_0 = [(x'-x'_0)^2 + y^2]^{1/2}$, $\theta_0 = \tan^{-1}[y/(x'-x'_0)]$, and p is the largest extension in the y -direction of the plastic zone. u^e and v^e are the displacements found by assuming linear elasticity throughout.

Further the stress at the boundary of the process region is

$$\sigma_{\theta} = s(v) \quad \text{for } 0 \leq x' < d, \quad y = 0 \quad (26)$$

and the remaining boundary conditions are

$$\sigma_y = 0 \quad \text{for } -a \leq x' < 0, \quad y = 0 \quad (27)$$

$$\tau_{xy} = 0 \quad \text{for } -a \leq x' < a, \quad y = 0 \quad (28)$$

$$u = 0 \quad \text{for } d \leq x' \leq a, \quad y = 0 \quad (29)$$

The region is covered by 450 elements as shown in Fig. 7. Load is applied in one single step in a procedure similar to the one for self-similar problems. Also the procedure for the modelling of the process region is similar to the one used for self-similar problems. Here, however, neither σ_D nor v_* can be specified a priori, but some values σ_D' and v_*' are obtained as results of the computation. By reformulating the problem repeatedly with the yield stress $\sigma_Y' = \sigma_Y \sigma_D' / \sigma_D$ where σ_D' is taken from the previous problem, a convergent solution is obtained after updating the yield stress 2 - 4 times. The solution of the original problem is found by scaling, as described for the self-similar problem.

The boundary conditions (24) and (25) can only be approximately satisfied. The residual forces, calculated at the iterations for equilibrium are given special attention at the trailing edge of the wake region, i.e. at $x' = -a$, $0 \leq y \leq p$. Near the edge the plastic strains are assumed to be uniformly

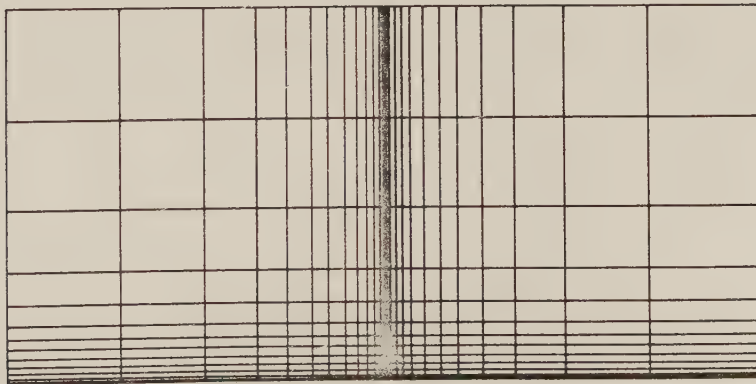


Figure 7. Principal element mesh for the steady-state problem.

distributed with respect to the x-direction and the elastic strains are assumed to be fairly small compared to these. Then residual forces are approximately constant along lines parallel to the x-axis, except for those acting on nodes at the very edge. The residual forces acting at the edge result in large deviations from (24) and (25). Hence, these forces are simply removed and replaced by nodal forces corresponding to the ones found at the first nodes immediately ahead, on lines $y = \text{constant}$. Since the starter displacement field is the one for a linearly elastic material, (24) and (25) are satisfied, whereas generally $\partial u/\partial y \neq \partial u^e/\partial y$ and $\partial v/\partial y \neq \partial v^e/\partial y$.

By introducing the fracture toughness K_{IC} the stress intensity factor K_I at steady-state can be written

$$K_I = K_{IC} = \alpha [v_* E \sigma_D / (1 - v^2)]^{1/2}$$

(30)

where α is given in Table 2.

The energy dissipated per unit of crack extension equals $(1 - v^2) K_{IC}^2 / E = \alpha^2 v_* \sigma_D$ and can be compared to the energy $v_* \sigma_D$ dissipated in the process region. Obviously the results show that more than half the total energy dissipation occurs in the process region. The plastic zones are shown in Figs. 8-9. The results for materials C and D are observed to agree well with the results obtained by Parks et al. [14], who, however neglected the existence of a process region.

As in the case of self-similarity one observes that the hardening rate has

Table 2.

Numerical factor α and non-dimensional process region length $\sigma_Y^2 d / K_{IC}^2$ for the four materials at steady state.

	α	$\sigma_Y^2 d / K_{IC}^2$
A	1.12	0.083
B	1.12	0.083
C	1.27	0.020
D	1.31	0.012

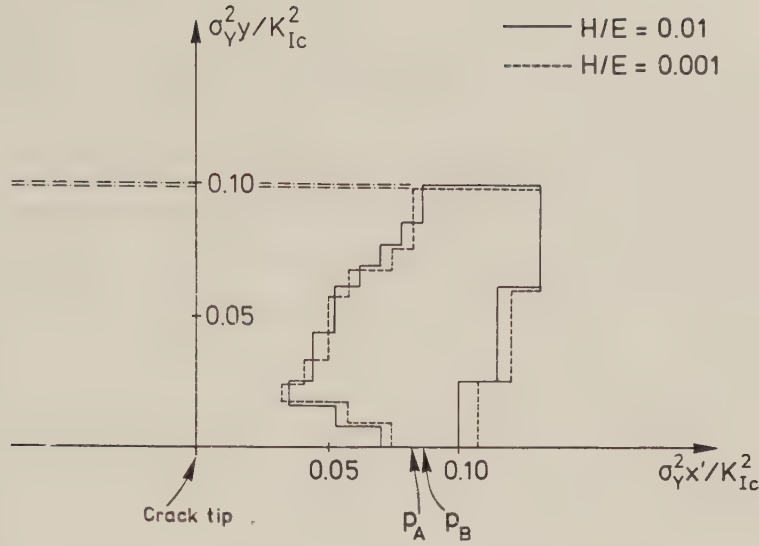


Figure 8. The shape of the plastic zones at steady-state for $\sigma_D/\sigma_Y = 2.5$. Material A with $H/E = 0.01$ (full lines) and material B with $H/E = 0.001$ (dotted lines). The position of the tip of the process region is at p_A for material A and at p_B for material B.

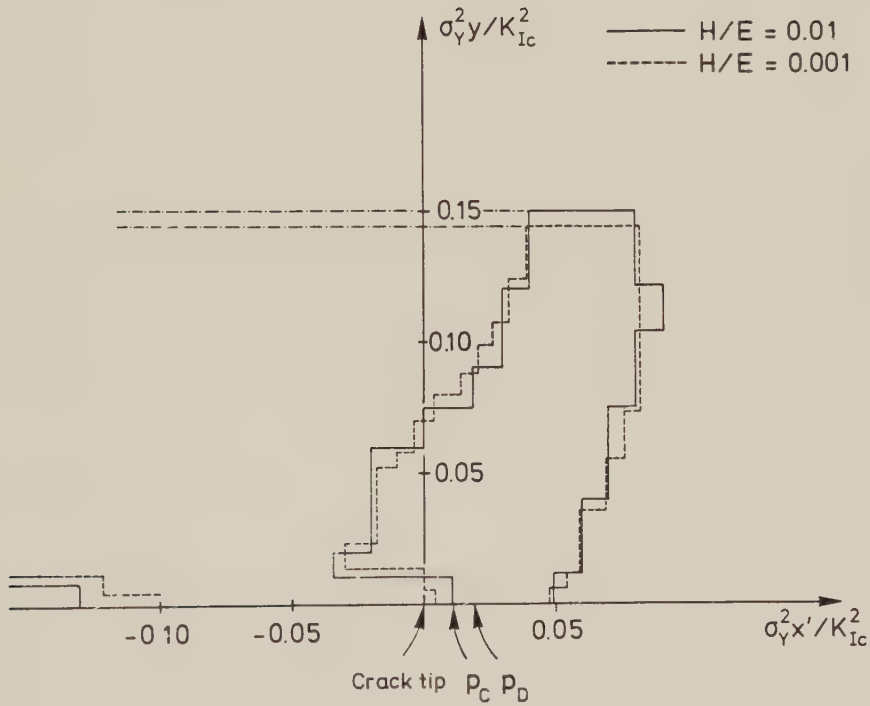


Figure 9. The shape of the plastic zones at steady-state for $\sigma_D/\sigma_Y = 3$. Material C with $H/E = 0.01$ (full lines) and material D with $H/E = 0.001$ (dotted lines). The position of the tip of the process region is at p_C for material C and at p_D for material D.

little influence on the the plastic region whereas the influence of the process region characteristics is remarkable. Reversed plasticity is found in the plastic region for materials C and D but not for A and B. The process region is well embedded in the plastic region for materials C and D but for materials A and B the length of the process region is much larger than the forwards extension of the plastic zone and is partly situated in the wake behind the plastic zone.

The differences in α for the four materials are interesting. It is quite obvious that materials C and D should possess larger fracture toughness than materials A and B because the process region can carry a maximum load $\sigma_D = 3\sigma_Y$ for C and D whereas $\sigma_D = 2.5\sigma_Y$ for A and B. Therefore one would perhaps anticipate that the ratio of the fracture toughness between the two groups of materials would be about $(3/2.5)^{1/2} = 1.1$. However the ratio is 1.24 - 1.28, cf. (30) and Table 2. This reflects, probably, an 'embrittling' influence of the process region recognizable from the fact that it is large compared to the linear extension of the plastic region for materials A and B.

6. Developing process region and stably growing crack

Due to the almost identical results for materials A and B in the last sections, material B is not investigated any further. Material D is also excluded. The reason is the relative uncertainty of the results due to the smallness of the process region. It was not possible to refine the element mesh as much as would be desirable, at least not within a reasonable computer cost.

Large cracks

The crack is assumed to be large enough that a length a can be found satisfying the inequalities (10). Thus the crack is assumed to be much larger than the lower limit according to the ASTM convention for linear fracture mechanics [15]. The boundary conditions (11), (12), (14) - (16) are used, whereas (13), specifying the stress at the boundary $0 \leq r < d$, $\theta = 0$, is replaced with

$$\sigma_\theta = s(v) \quad (31)$$

where $s(v)$ is given by (7). The area $r \leq a$, $0 \leq \theta \leq \pi$ is covered by 136 elements as shown in Fig. 10. The load is applied in small increments. Thus

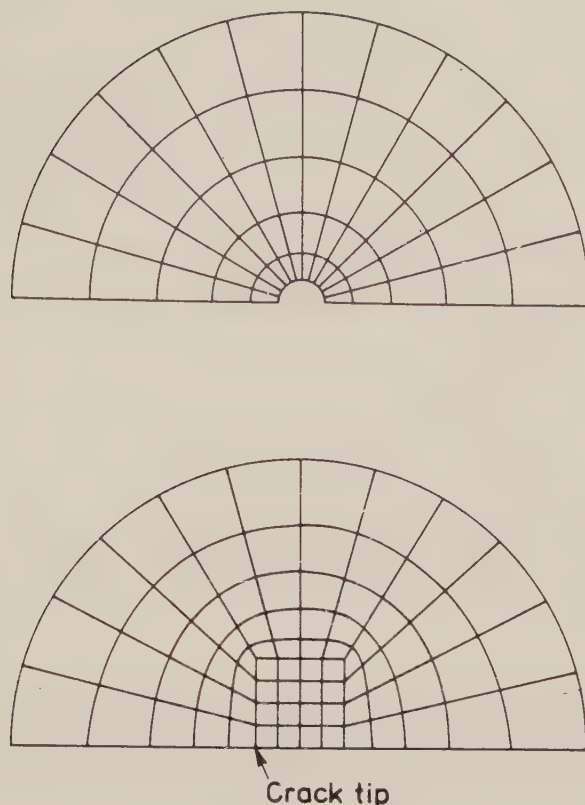


Figure 10. Element mesh for the large crack problem. The part around the crack tip is enlarged below.

about 20-30 increments were needed to fully develop the process region, and then additionally about 40-50 ones to reach global instability.

The results are displayed in Fig. 11 as the non-dimensional x-positions of the tip of the process region $\sigma_Y^2(c+d-c_0)/K_{IC}^2$ and the tail $\sigma_Y^2(c-c_0)/K_{IC}^2$ versus non-dimensional squared stress intensity factor $(K_I/K_{IC})^2$. The fracture toughness K_{IC} is assumed to be the one found from the special investigation for the steady-state case.

Apparently due to the fact that condition (10) could not be satisfied as well as would be desirable, global instability appeared to be approached at a slightly lower load level than found for the steady-state case. Global instability conditions seem to be approached rather slowly for material A and somewhat faster for material C. Onset of stable crack growth occurred at about 64% of instability load for material A and at about 72% for material C.

Plastic zones are shown in Figs. 12-13 at the onset of crack growth and immediately below instability load. After crack growth of about the forwards extension of the plastic zone at onset of stable crack growth, one observes that the shape of the active part (with primary plastic loading) of the plastic

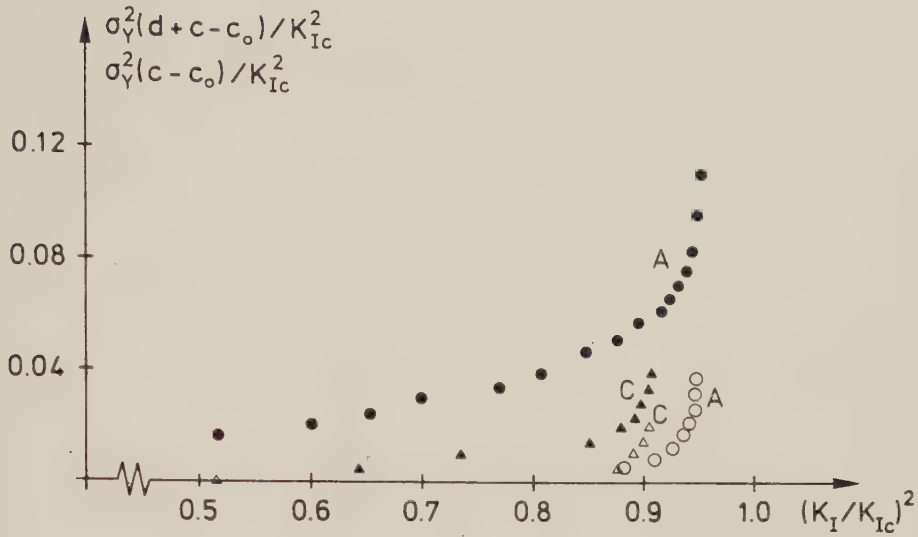


Figure 11. Non-dimensional x-position of the tip $\sigma_Y^2(d+c-c_o)/K_{Ic}^2$ (dark symbols) and the tail $\sigma_Y^2(c-c_o)/K_{Ic}^2$ (light symbols) of the process region versus non-dimensional squared stress intensity factor K_I/K_{Ic} . Onset of crack growth occurred at $(K_I/K_{Ic})^2=0.41$ for material A at $(K_I/K_{Ic})^2=0.52$ for material C.

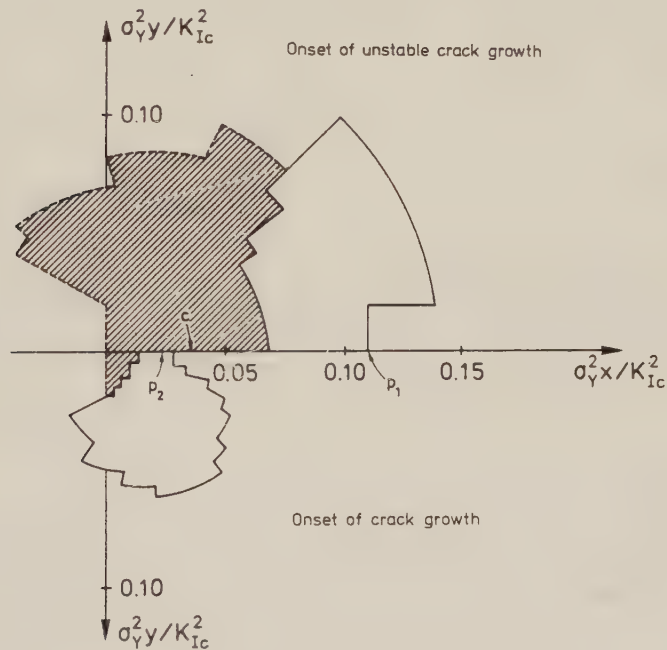


Figure 12. The shape of the plastic zones for large cracks at onset of crack growth and immediately below instability load for material A. The position of the tip of the process region is at p_1 at onset of crack growth and at p_2 just before instability and c is the position of the crack tip just before instability. The shaded areas are wake regions.

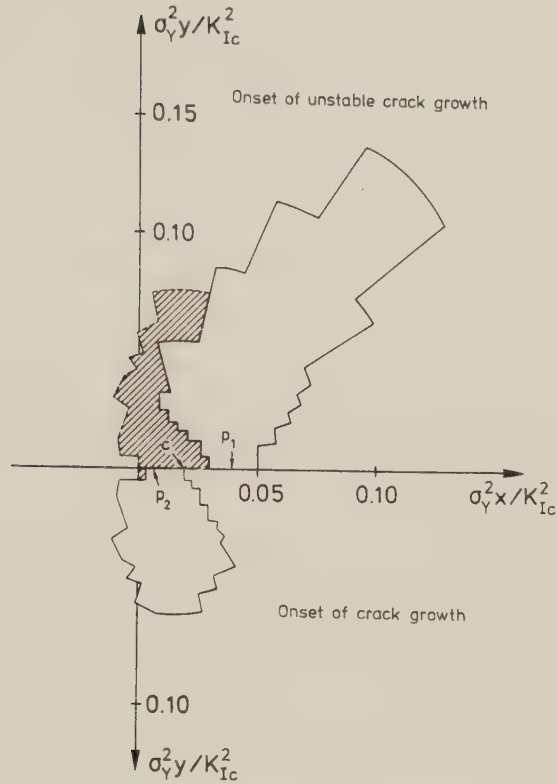


Figure 13. Same as Fig. 12 for material C.

zone resembles the shape at steady-state.

Intermediate and small cracks.

According to the ASTM convention, see Srawley and Brown [15], the limit of linear fracture mechanics is specified through

$$c_o \geq 2.5(K_{Ic}/\sigma_y)^2 \quad (32)$$

In this section cracks whose lengths range from 0.1 to 1.0 times c_{ASTM} , the crack length at the ASTM-limit are studied. K_{Ic} -values for the steady-state case are used. A region $0 \leq x \leq a$, $0 \leq y \leq a$ is considered. a is chosen to $10c_o$. The following boundary conditions are used:

$$\sigma_y = \sigma_\infty \quad \text{for } 0 \leq x \leq a, y = a \quad (33)$$

$$\tau_{xy} = 0 \quad \text{for } 0 \leq x \leq a, y = a \quad (34)$$

$$\sigma_x = 0 \quad \text{for } x = a, 0 \leq y \leq a \quad (35)$$

$$\tau_{xy} = 0 \quad \text{for } x = a, 0 \leq y \leq a \quad (36)$$

The remaining boundary conditions are (1)-(6) and the stress at the process region is specified by (7). The region studied is covered by 230 elements as shown in Fig. 14.

The results for the crack lengths $c_o = c_{ASTM}$, $c_o = 0.5c_{ASTM}$ and $c_o = 0.15c_{ASTM}$ are shown in Figs. 15-17. The amount of stable crack growth decreases with decreasing crack length but the relative amount (i.e. in relation to the crack length) increases. It is possible that the relative amount would decrease for smaller cracks than those investigated here, and that instability occurs even before the process region is fully developed in the case of very small cracks, cf. [6].

The length of the process region at stable crack growth is considerably smaller for material C than for material A, in agreement with the results for

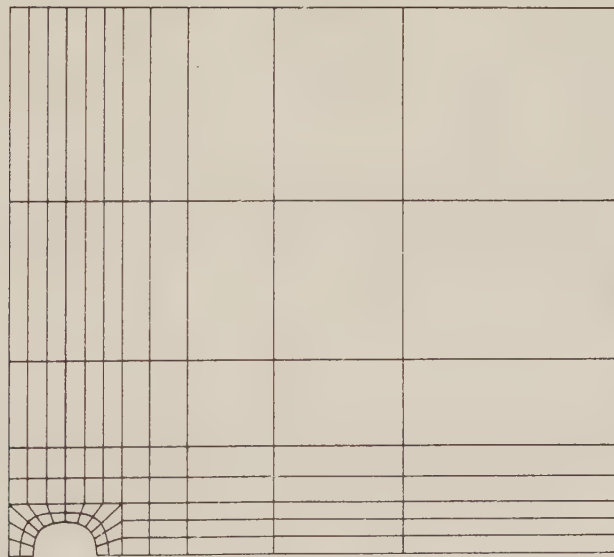


Figure 14. Element mesh for intermediate and small cracks. The part around the crack tip is the mesh shown in Fig. 10.

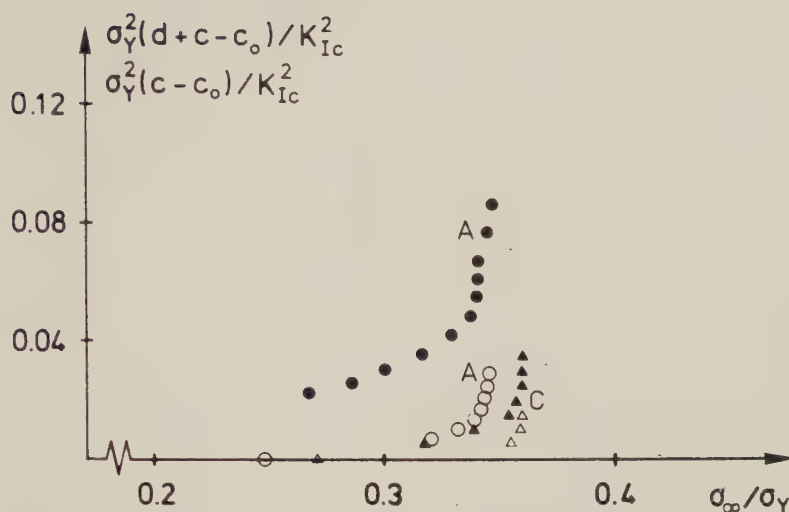


Figure 15. Non-dimensional x-position of the tip $\sigma_Y^2(d+c-c_o)/K_{Ic}^2$ (dark symbols) and the tail $\sigma_Y^2(c-c_o)/K_{Ic}^2$ (light symbols) of the process region versus non-dimensional load σ_∞/σ_Y for a crack length at the ASTM-limit.

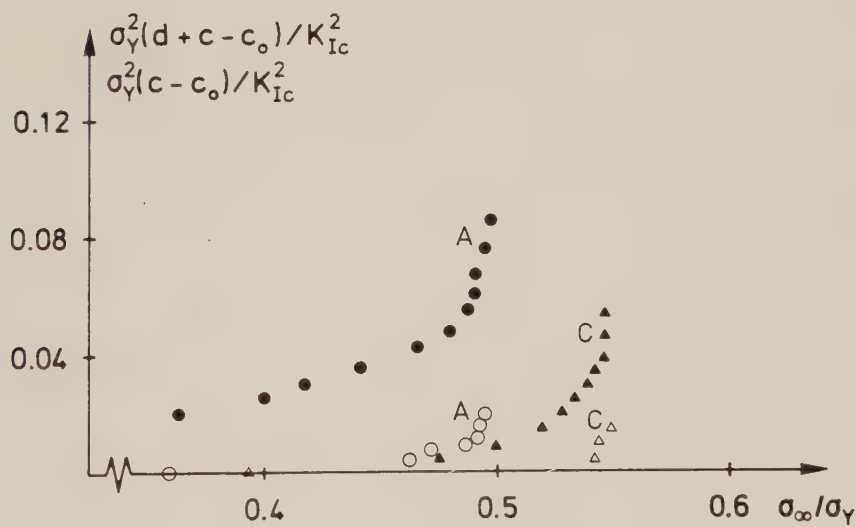


Figure 16. Same as Fig. 15 for a crack length $c_o = 0.5c_{ASTM}$.

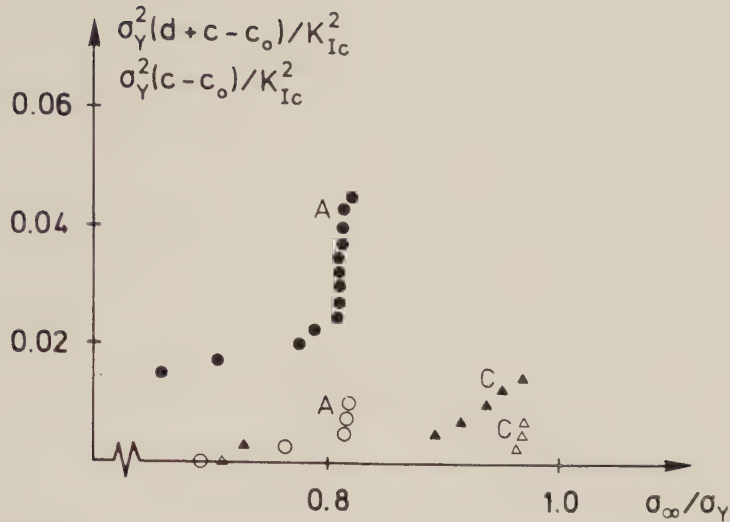


Figure 17. Same as Fig. 15 for a crack length $c_o = 0.15c_{ASTM}$.

large cracks, see Table 2. The length of the process region increases several times during the early stages of crack growth. However, there is some uncertainty as to the accuracy of the load at onset of crack growth. The error should although be 10% at most. For the smallest cracks studied onset of unstable crack growth occurs at a remote load which is not very far from the one for general yield in a plate without a crack, i.e. about $1.12\sigma_Y$.

Figure 18 shows the remote load at instability as a function of the original length of the crack. The results for crack lengths $c_o = 0.7c_{ASTM}$, $c_o = 0.3c_{ASTM}$ and for material A $c_o = 0.1c_{ASTM}$ are included as well. Due to stability reasons it was not possible to compute material C for the smallest crack length. It is seen that material C shows a higher fracture strength than material A. This might seem to be logical in view of the fact that the process region 'strength' is higher for material C. However, the representation in Fig. 18 concerns remote load at instability versus the crack length divided by the crack length, c_{ASTM} , at the ASTM-limit. Differences in the fracture toughness are therefore already considered. Hence, linear fracture mechanics is represented by one single curve in the diagram (actually included for comparison) irrespective of differences in fracture toughness. The explanation of the lower strength of material A should rather be the presumed 'embrittlement' discussed for the steady-state case.

In Figs. 19-22 plastic zones are shown for two crack lengths $c_o = 0.5c_{ASTM}$ and $c_o = 0.15c_{ASTM}$. At onset of instability the zones are smaller in material A than in material C at the same relative crack length.

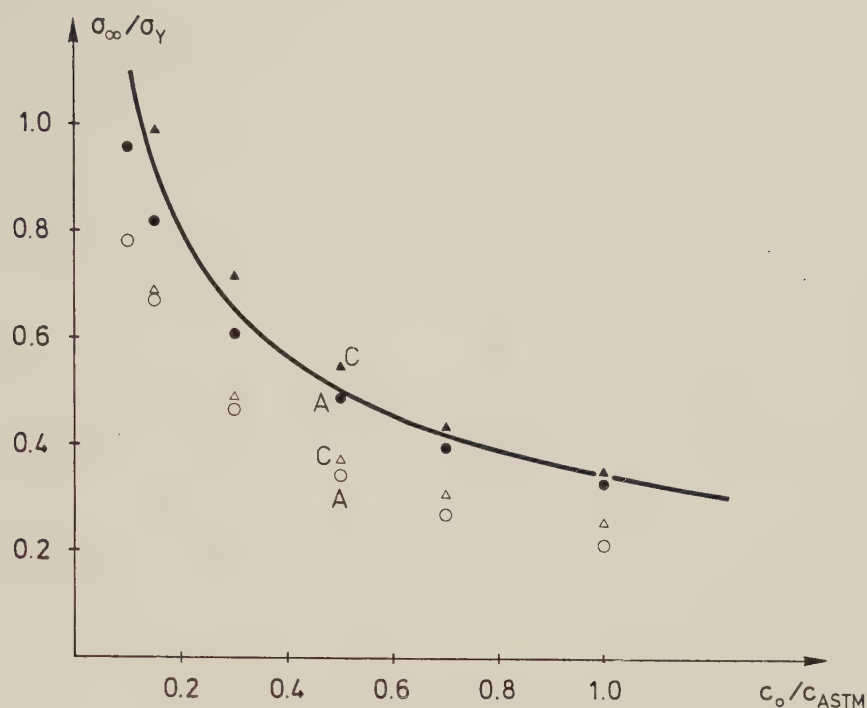


Figure 18. Non-dimensional remote load σ_{∞}/σ_Y at onset of crack growth (dark symbols) and at instability load (light symbols) versus crack length divided with the crack length at the ASTM-limit, c_0/c_{ASTM} . $\bullet$ $\circ$ for material A and $\blacktriangle$ $\triangle$ for material C.

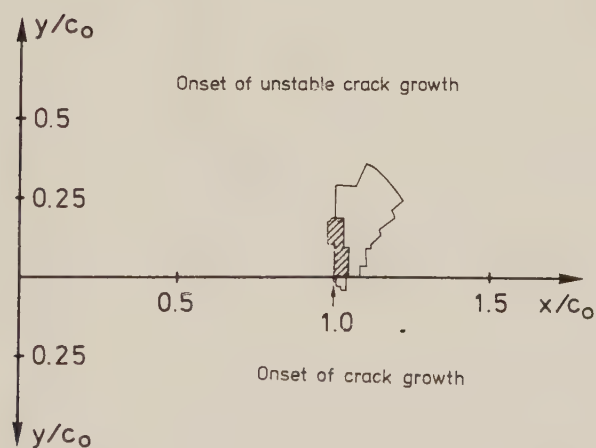


Figure 19. The shape of the plastic zones at onset of crack growth and immediately below instability load for a crack length $c_0 = 0.5c_{ASTM}$ for material A. The shaded areas are wake regions.

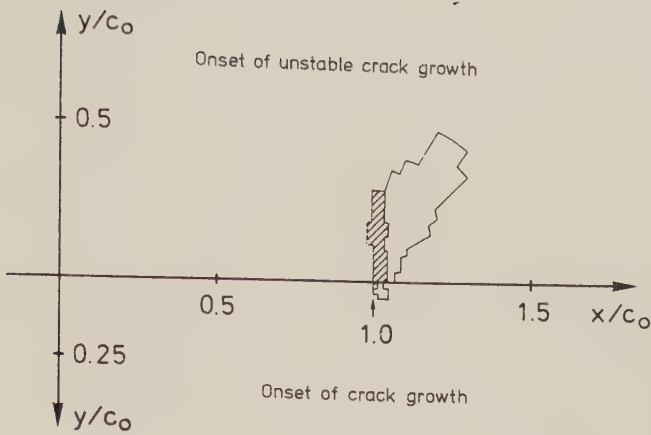


Figure 20. Same as Fig. 19 for material C.

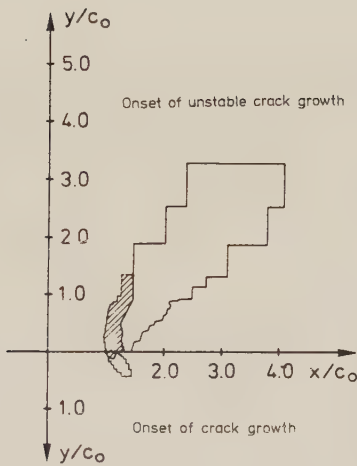


Figure 21. The shape of the plastic zones at onset of crack growth and immediately below instability load for a crack length $c_0 = 0.15c_{ASTM}$ for material A.

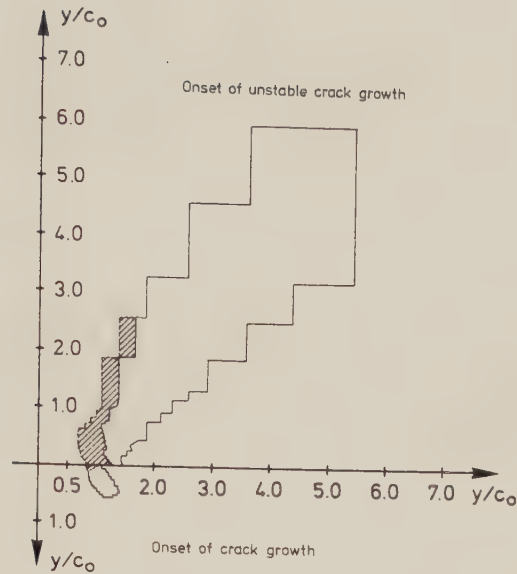


Figure 22. Same as Fig. 21 for material C.

6. Discussion

The role of the process region at onset of crack growth, during stable crack growth and at steady-state was found to be much more important than the role of the hardening rate. Even for the fairly small variation of the cohesive strength - from $\sigma_D = 2.5\sigma_Y$ to $\sigma_D = 3\sigma_Y$ - considerable differences appear.

The lower cohesive strength seems to impede the development of the plastic region. The process region almost pierces the plastic region whereas it is well embedded in the plastic region at the higher cohesive strength. The sizes and shapes of the plastic regions therefore become remarkably different. It is little surprise that the plastic region for a low load (in the self-similarity range) agrees well with the one found by neglecting the process region completely for the higher cohesive strength but not for the lower cohesive strength. For a low hardening material the length of the process region was found to approach the one for the Dugdale model. This seems logical due to the relative smallness of the plastic zone compared to the length of the process region.

The results obtained show that the influence of the cohesive strength on the fracture toughness is considerably more pronounced than one would anticipate from the direct contribution of the cohesive strength. This seems

to be explained by an 'embrittling' influence of the process region at low cohesive strength. It was also observed that more than half the energy dissipation at steady-state occurs in the process region at the lower cohesive strength in fact as much as 80%.

ACKNOWLEDGEMENTS

I would like to thank Professor K. B. Broberg for numerous invaluable discussions and encouraging support during the course of this work. I would also like to thank Mr A. Lundström for critical comments on the manuscript.

REFERENCES

- [1] K. B. Broberg, Journal of the Mechanics and Physics of Solids 23 (1975) 215-237.
- [2] G. I. Barenblatt, Advances in applied Mechanics 7, Academic Press, New York (1962) 55-129.
- [3] H. Andersson, Journal of the Mechanics and Physics of Solids 21 (1973) 337-356.
- [4] A. P. Kfoury and K. J. Miller, International Journal of Pressure Vessels and Piping 2 (1974) 179-189.
- [5] A. Lundström, International Journal of Fracture 21 (1983) 123-132.
- [6] P. Stähle, International Journal of Fracture 22 (1983) 203-216.
- [7] G. C. Nayak and O. C. Zienkiewicz, International Journal of Numerical Methods in Engineering 5 (1972) 113-134.
- [8] P. Stähle, Report from the Division of Solid Mechanics, TFHF-3015, Lund Institute of Technology, Lund, Sweden (1985).

- [9] M. L. Williams, Journal of Applied Mechanics 24 (1957) 109-114.
- [10] H. Tada, P. Paris and G. Irwin, The Stress Analysis of Cracks Handbook, Del Research Corp., Hellertown (1973).
- [11] D. S. Dugdale, Journal of the Mechanics and Physics of Solids 8 (1960) 100-104.
- [12] J. R. Rice, Journal of Applied Mechanics 35 (1968) 379-389.
- [13] N. Levy, P. V. Marcal, W. J. Ostergren and J. R. Rice, International Journal of Fracture Mechanics 7 (1967) 143-156.
- [14] D. M. Parks, P. S. Lam and R. M. McMeeking, Advances in Fracture Research 4, Pergamon Press, New York (1981) 2607-2614.
- [15] W. F. Brown and J. E. Srawley, Plane Strain Crack Toughness Testing of High Strength Metallic Materials, ASTM Special Technical Publication 410 (1966).

